

Consider the following descriptions of functions  $f_1(x), \dots, f_6(x)$ :

1.  $f_1'(x) > 0$  for  $2 \leq x < 7$ ,  $f_1(2) = 3$ , and  $f_1(7) = 3$ .
2.  $f_2(2) = 3$ ,  $f_2(6) = 7$ , and for  $2 \leq x \leq 6$ ,  $f_2'(x) \leq 0$ .
3.  $f_3'(x)$  exists and is  $\neq 0$  for  $0 \leq x \leq 4$  and  $f_3(0) = -5$  and  $f_3(4) = -5$ .
4.  $f_4'(x) > 0$  for  $2 \leq x \leq 7$ ,  $f_4(2) = 10$ , and  $f_4(7) = 3$ .
5.  $f_5'(x) = 0$  for all  $x$  except  $x = 3$ ,  $f_5(1) = 7$ ,  $f_5(5) = 2$ .
6.  $f_6'(x)$  exists and is  $\neq 0$  for  $0 \leq x < 2$  and  $2 < x \leq 4$ ; and  $f_6(0) = -5$  and  $f_6(4) = -5$ .

For each  $f_n(x)$  ( $1 \leq n \leq 6$ ):

- Determine if such a function  $f_n(x)$  can possibly exist. (Three are possible, three are impossible. Remember that if  $f_n'(c)$  exists, then  $f_n$  must be continuous at  $x = c$ , but at points where  $f_n'(c)$  does not exist,  $f_n(x)$  may not be continuous.)
- If such an  $f_n(x)$  can exist, draw the graph of one possible  $f_n(x)$ .
- If  $f_n(x)$  can't possibly exist, explain why (using the Mean Value Theorem), and get your group to agree on the explanation.